Identification of Multilayer Soil Models for Grounding Systems from Surface Measurements

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***Abstract* -- Soil modelling is one of the most crucial aspects in grounding system analysis. A large body of literature concerning methods for obtaining the model parameters by means of suitable inversion techniques starting on resistivity survey data exists. In general, the soil resistivity changes both in horizontal and vertical direction but simulations for engineering applications require a simplified model, i.e. uniform, multilayer or multizone depending on the specific situation and system size. The multilayer soil model is often the most suitable for grounding system analysis. The number of required layers is in general site-dependent and on the basis of literature, the simplistic double layer soil model is adequate only in about the 20% of cases while in the remaining 80% of cases a model with three or more layers is required.**

This work focuses on the identification of multilayer soil model parameters starting from surface resistivity measurements. The proposed methodology is based on an optimization approach. It is shown that the problem is ill-posed in general and that in practical cases it is only possible to find a solution which is accurate enough for engineering purposes.

The paper is based on simulations performed by the soil resistivity analyzer implemented in a commercial simulation environment. A distinctive feature of this work is that it tries to bridge a gap between the well-studied mathematical theory of the Calderon problem and the equally well-established engineering approach.

***Index Terms*-- Grounding Systems, Earthing Systems, Multilayer Soil Models, Inversion Methods**

I. INTRODUCTION

Soil modeling represents one of the most crucial aspects in grounding system analysis. Indeed, a large body of literature concerning the criteria required to set up an appropriate soil model which can be used to predict the performances of a grounding system (see [1] for instance) exists.

In physical reality the soil resistivity changes arbitrarily in space, i.e. it is represented by the function

$$\boldsymbol{\rho} = \boldsymbol{\rho}(x, y, z). \tag{1}$$

If anisotropy is negligible, the resistivity becomes a scalar function:

$$\rho = \rho(x, y, z). \tag{2}$$

The soil structure and thus the soil resistivity in general change both in vertical and horizontal in a rather unpredictable way direction (see Fig. 1.1).

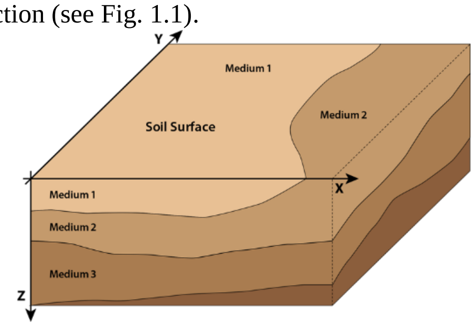


Fig. 1.1. A typical soil cross section

The real 3D distribution of the soil resistivity is difficult to determine since the limited data available for the solution of the inverse problem are collected by means of surface surveys. Modern inversion techniques allow to obtain 1D, 2D and also 3D distributions of the soil resistivity.

From a theoretical point of view, the inverse problem has a unique solution for bounded domains, but only under strict conditions which require that the voltage and current distributions are continuous and precisely known over the whole boundary for a complete set of current injection patterns [2], as discussed in detail in Section II..

In practice, the problem of obtaining the resistivity distribution from boundary data is ill-posed since such data is collected by a finite number of electrodes covering only part of the surface. Moreover, measurement uncertainty further complicates the problem. This implies that more than one model will produce responses consistent with the observed data [3]. Due to these considerations, simplified models are typically used in the engineering practice: 1D models are commonly adopted for grounding systems while 2D and 3D soil models are applied to groundwater investigations, civil engineering or environmental surveys.

For systems of the size of typical grounding installations, the vertical changes in soil resistivity are usually predominant with respect to the horizontal ones, thus justifying the use of simplified 1D models.

With this representation of the resistivity distribution, the inverse problem becomes well-posed if the voltage and current distributions are continuous and exactly known along an arbitrary infinitely long line on the soil surface.

If on the other hand the voltage and current distributions are known only for a finite number of points the problem is once again ill-posed, i.e. in practical cases the inverse problem has many or an infinite number of equivalent solutions. Among these solutions, the choice of a specific distribution should be based on geological plausibility, which imposes physical constraints which must be taken into account.

Typical geologically plausible simplified scenarios include a uniform and multilayer soil cases.

A uniform soil model should be used only when there is a moderate variation in measured data, which is not the case for the majority of soils. A uniform soil model may also be used at high frequency because in that case, the skin effect limits the penetration depth of the electromagnetic field to a few meters and therefore the soil resistivity of the deep layers does not affect the results.

In the case of small systems, i.e. grounding systems with a size up to a few hundred meters, the soil model is not significantly affected by horizontal changes in soil resistivity and therefore a multilayer soil model is usually the most appropriate choice. The number of layers required by the model to accurately represent measured data depends on the actual soil resistivity variations in the vertical direction (which may also depend on seasonal effects like frozen soil layers) and three, four or more layers may be required to obtain sufficient accuracy.

In the case of systems of intermediate size, i.e. grounding systems with a size up to a few kilometers, the soil model is affected by both horizontal and vertical changes in soil resistivity and usually an equivalent double or triple layer soil model is chosen.

In the case of large systems, i.e. those with a maximum size over a few kilometers, where the soil surface is often not even flat, the soil model is significantly affected by horizontal changes in soil resistivity and usually a multizone soil model must be adopted. The number of zones depends on the systems size and soil resistivity variations in the horizontal direction.

From the above, the multilayer soil model represents very often the most appropriate way to represent the resistivity distribution in the soil for the sake of grounding system analysis and this paper will focus on this approach.

The rest of the paper is organized as follows: the first part of the paper describes the theoretical approach used to obtain the parameters of a multilayer soil model starting from surface measurements. Then, some validation cases are discussed and some practical limitations of the common Wenner and Schlumberger identification approaches are highlighted. Finally a case study based on real measurements is presented together with practical indications on how to correctly interpret the results of a single set of measurements. Unless otherwise noted all quantities are in SI units.

II. Mathematical Theory

A large body of mathematical literature is concerned with the so-called Calderón's inverse conductivity problem [4], also known as Electrical Impedance Tomography problem. The problem, without delving into mathematical details, consists in identifying the resistivity distribution starting from boundary data, with very strong analogies with the problem object of this study. However, the mathematical study of the problem analyses in extreme detail and rigor the regularity characteristics of the admissible resistivity function, the boundary, and the boundary data (potential and current) [5], [6].

It is well known that the mathematical problem is in general severely ill-posed [7], however it has been proven that if it is a-priori known that the conductivity is piecewise constant with a bounded number of unknown values, like in the case of multilayer soil models, then a Lipschitz stability estimate holds [8]. This does not mean that the engineering approach described in the next Section is capable of solving the mathematical problem for three main reasons: 1) the practically available boundary data are limited, 2) the real resistivity distribution does not match the multilayer hypothesis, 3) measurement data are affected by errors. In spite of these differences it is to be expected that the more the physical reality and the measurements approach the hypothesis of the proven theorems, the higher will be the probability of correctly identifying the model parameters. Indeed, it has been proven in [9] that for the case of a multilayer model an algorithm based on limited, but exact, boundary data similar to the one prosed in the next Section will converge to the correct solution of the problem as the amount of available data increases.

III. Engineering Approach

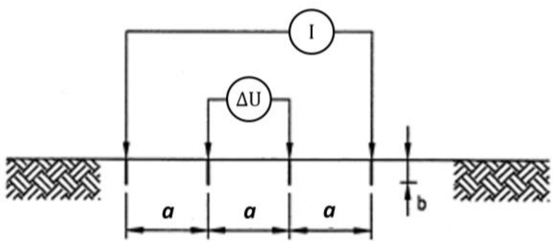
The electromagnetic properties of the soil include resistivity, permittivity and permeability (normally considered as equal to the vacuum permeability).

The soil resistivity and permittivity depend on moisture, temperature and chemical composition, and are also frequency dependant. There is no generally accepted formulation to express such dependency and several models exist ([10], [11], [12]). At power frequency (50 Hz and 60 Hz) the effects of the permittivity of the soil can be safely neglected and the soil resistivity can be approximated with its DC value. Thus, in the following the soil parameters will be considered frequency independent and the DC soil resistivity will be adopted.

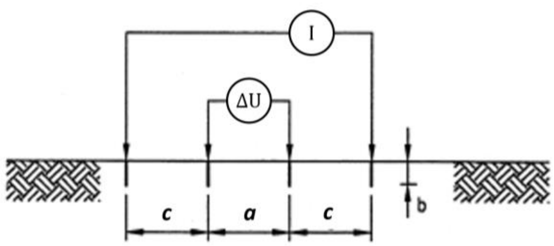
The soil resistivity value may vary over several orders of magnitude, extending from low values in the order of 1-10 Ωm, to high values in the order of 1000-10000 Ωm. Typical values range from 10 Ωm up to 1000 Ωm.

Due to the complexity of the soil composition and the dependence on local factors, including environmental ones, it is not possible to assign a single value to the resistivity of a soil material. For this reason, in the case of grounding system applications, resistivity values cannot be reliably obtained from literature but must be estimated from data measured at the specific site.

Soil model parameters are typically obtained starting from measured data collected with the Wenner and/or Schlumberger four-pin methods. These methods (see Fig. 2.1) represent the most commonly used techniques for soil resistivity measurement.



Wenner



Schlumberger

Fig. 2.1. Four-pin methods for soil resistivity measurements

The techniques used for soil resistivity measurements are named lateral profiling and vertical sounding. In lateral profiling, the relative positions of the four electrodes are kept fixed and the four electrodes are moved together along the survey line. In vertical sounding, on the other hand, the center point of the electrodes array remains fixed but the spacing between the electrodes is changed. With the increase of the spacing, information about the deeper sections of the subsurface is obtained.

In the following only vertical sounding will be treated. This technique does not allow to appreciate lateral changes in resistivity but this is not an issue in the assumption of a multilayer (and not multizone) soil model.

If the electrode probe diameter is not more than 10% of the distance between them, using the Wenner method the apparent soil resistivity value is [13]:

$$\rho_E = \frac{4\pi a R_W}{1+\dfrac{2a}{\sqrt{a^2+4b^2}}-\dfrac{a}{\sqrt{a^2+b^2}}} , \qquad (3)$$

where a is the electrode spacing, b is the probe depth and R_w is the Wenner resistance measured as ΔU/I in Fig. 2.1. If $b << a$, (3) simplifies to:

$$\rho_E = 2\pi a R_W . \qquad (4)$$

If the electrode probe diameter is not more than 10% of the distance between them, using the Schluberger method the apparent soil resistivity value is [13]:

$$\rho_E = \frac{2\pi R_S}{\dfrac{1}{c}-\dfrac{1}{a+c}+\dfrac{1}{\sqrt{c^2+4b^2}}-\dfrac{1}{\sqrt{(a+c)^2+4b^2}}} , \qquad (5)$$

where a is the spacing between voltages probes, b is the probes depth, c is the spacing between voltage and current probes and R_s is the Schluberger resistance measured as ΔU/I in Fig. 2.1. If $b << a$ and $b << c$, (5) simplifies to:

$$\rho_E = \pi \frac{c(c+a)}{a} R_S . \qquad (6)$$

The apparent resistivity depends explicitly on known experimental parameters (a, b, c) and implicitly on the soil composition and stratification (R_s or R_w) which influence the current penetration in the soil.

Using a suitable set of electrode spacings it is then possible to investigate how resistivity changes with depth.

As mentioned, in general the problem of obtaining soil model parameters stating from resistivity survey data (inverse problem) is ill-posed, especially in the case of data obtained according to the Wenner or Schlumberger approach.

In the past this problem was solved graphically using auxiliary curves, while nowadays numerical approaches are preferred.

The essential theory is included in [14].

Using Wenner's sampling method and the simplified formula (4), the apparent soil resistivity can be calculated as follows:

$$\rho_c(a) = 2\pi a \frac{\Delta U}{I} = 4\pi a \frac{U(a)-U(2a)}{I} . \qquad (7)$$

The potential created on the soil surface by a pointwise current source located on the soil surface can be calculated as

$$U(a) = \frac{\rho_1 I}{2\pi}\int_0^{\infty}\left[1+2B(\lambda)\right] J_0(\lambda a)\, d\lambda , \qquad (8)$$

where a is the distance between source and calculation point, I is the point current, ρ_1 is the soil resistivity of the upper layer, $J_0(\lambda a)$ is the Bessel function of first kind and zero order and $B(\lambda)$ is the so called kernel function which depends on the number of layers. For a five layer soil model, considering the deepest layer of infinite extension, the kernel function reads [14]:

$$B_5(\lambda) = \frac{K_{51} e^{-2\lambda h_1}}{1-K_{51} e^{-2\lambda h_1}} , \qquad (9a)$$

$$K_{51} = \frac{\upsilon_{12} + K_{52} e^{-2\lambda h_2}}{1 + \upsilon_{12} K_{52} e^{-2\lambda h_2}}, \qquad (9b)$$

$$K_{52} = \frac{\upsilon_{23} + K_{53} e^{-2\lambda h_3}}{1 + \upsilon_{23} K_{53} e^{-2\lambda h_3}}, \qquad (9c)$$

$$K_{53} = \frac{\upsilon_{34} + \upsilon_{45} e^{-2\lambda h_4}}{1 + \upsilon_{34} \upsilon_{45} e^{-2\lambda h_4}}, \qquad (9d)$$

$$\upsilon_{ij} = \frac{\rho_j - \rho_i}{\rho_j + \rho_i}, \qquad (9e)$$

where ρ_i and h_i are the resistivity and the thickness of the i-th layer, respectively. With (7) and (8), the formula for the apparent soil resistivity becomes:

$$\rho_c(a) = \frac{\rho_1}{2\pi a} + 2\frac{\rho_1}{\pi}\int_0^\infty B(\lambda)\left[J_0(\lambda a) - J_0(2\lambda a)\right] d\lambda \,. \qquad (10)$$

The solution of the inverse problem allows to calculate the soil parameters of a multilayer soil model with an arbitrary number of layers *n* [15], [16]. Starting from an arbitrary set of *N* measured values, the problem consists in finding the *2n-1* parameters that minimize the following squared error function:

$$\psi = \sum_{i=1}^{N} \left[\frac{\rho_m(a_i) - \rho_c(a_i, \rho_1, \rho_2, \cdots \rho_n, h_1, h_2, \cdots h_{n-1})}{\rho_m(a_i)} \right]^2 . \qquad (11)$$

where $\rho_m(a_i)$ are the measured apparent soil resistivity values, $\rho_c(a_i)$ are the calculated ones and a_i are the electrodes spacings.

The number of measured values *N* should be greater than the number of parameters *2n-1*.

As anticipated, the inverse problem has a unique solution only if the potential expressed by (8) is known for all values of *a*. Moreover the potential should be continuous.

Practically, the best one can hope for is to approximate this continuous potential with a very high number of high quality measurements. Moreover, only if the soil is really stratified in horizontal layers separated by flat horizontal interfaces (multilayer soil models), the error function (11) has a zero minimum value. In real cases, (11) may have one or more minima but with a value greater than zero. The error function (11) can be modified by applying suitable weights to the measured values with smaller and bigger electrodes spacing. This is usual and useful in practical cases [17].The minimization of (11) can be carried out using zero order methods (e.g. downhill simplex method, genetic algorithms) or higher order methods (e.g. steepest descent method, Levenberg-Marquardt method, conjugate gradient method, trust region method) [17]. Results reported here refer to the use of the constrained trust region method which was chosen because of its robustness, i.e. its applicability to ill conditioned problems. In general the result at which a minimization algorithm converges, which may be a local minimum instead of the global one, depends on the initial guess. This problem can be limited with many techniques, for example by using a suitable set of initial guesses or by appropriate restarting methods. Furthermore, constraints can be usefully applied in order to avoid meaningless solutions like those having unphysical resistivities or thicknesses, i.e. those not corresponding to plausible geological reality.

IV. Simulation Environment

Results presented in the following two sections were obtained with the SRA (Soil Resistivity Analysis) module implemented in the XGSLab® simulation environment which is based on the so called PEEC “Partial Element Equivalent Circuit” method.

The theoretical background on which XGSLab® is based can be found in [14], [13], [18], [19], [20], [21], [22].

In particular, the SRA module allows to use uniform, multilayer and multizone soil models (with an arbitrary number of layers and zones), although this latter aspect is not considered here.

Furthermore, XGSLab® can consider frequency dependent soil parameters according to different models ([10], [11], [12]) but in the following, taking into account the frequencies of interests, the soil parameters will be considered frequency independent and the low frequency soil resistivity will be adopted.

V. Validation

In this section, we validate the proposed methodology based on the application of a constrained trust region minimizer on some artificial multilayer soil models. The objective is to show that if a soil is indeed purely horizontally stratified and measurements are not affected by errors then the method is able to identify the layers’ properties with sufficient accuracy.

The apparent soil resistivity values (considered as measured values) shown in Table II have been calculated with (10) and refer to the soil models with 2, 3, 4 and 5 layers, whose parameters are shown in Table I.

TABLE I
Soil model parameters used for validation

	2 layers	3 layers	4 layers	5 layers
ρ_1	100.0	100.0	100.0	100.0
ρ_2	50.00	50.00	50.00	50.00
ρ_3	-	200.0	200.0	200.0
ρ_4	-	-	75.00	20.00
ρ_5	-	-	-	300.0
h_1	2.000	2.000	2.000	2.000
h_2	-	6.000	6.000	6.000
h_3	-	-	15.00	10.00
h_4	-	-	-	15.00

TABLE II

MEASURED APPARENT SOIL RESISTIVITY AS A FUNCTION OF THE ELECTRODES SPACING

a	2 layers ρ	3 layers ρ	4 layers ρ	5 layers ρ
1.000	97.59	97.65	97.65	97.93
1.500	93.54	93.74	93.74	93.89
2.000	88.27	88.75	88.73	88.80
3.000	77.50	79.00	78.95	78.88
4.000	69.01	72.27	72.15	71.91
5.000	63.17	68.89	68.65	68.16
7.500	55.83	70.00	69.23	67.70
10.00	53.08	77.24	75.49	72.26
15.00	51.26	94.69	89.56	81.36
20.00	50.69	110.0	99.75	85.75
30.00	50.30	132.6	108.7	85.53
40.00	50.17	147.8	109.0	82.88
50.00	50.11	158.5	105.8	82.32
75.00	50.05	174.5	95.58	91.03
100.0	50.03	182.9	88.13	106.1

Using this data, without applying weights to the measured values, the parameters calculated by the trust region minimizer are shown in Table III (see also Fig. 3.1).

TABLE III

SOIL MODEL PARAMETERS CALCULATED BY SRA

	2 layers	3 layers	4 layers	5 layers
ρ_1	100.0	100.0	100.0	100.3 (+0.30%)
ρ_2	50.00	50.00	50.03 (+0.060%)	50.37 (+1.1%)
ρ_3	-	200.0	200.7 (+0.35%)	223.6 (+12%)
ρ_4	-	-	75.07 (+0.093%)	26.19 (+31%)
ρ_5	-	-	-	302.5 (+0.83%)
h_1	2.000	2.000	1.999 (-0.05%)	1.981 (-0.95%)
h_2	-	6.002 (+0.033%)	6.012 (+0.20%)	6.219 (+3.6%)
h_3	-	-	14.89 (-0.73%)	8.496 (-15%)
h_4	-	-	-	20.02 (+33%)
RMS %	0.002893	0.00763	0.005795	0.008961

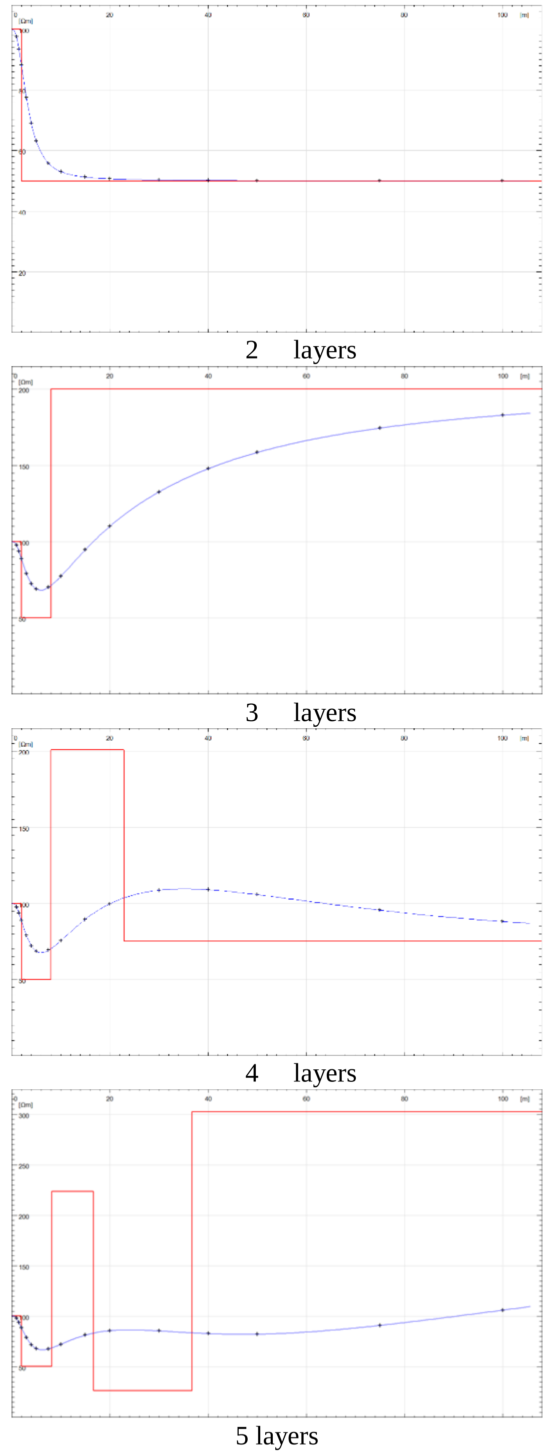


Fig. 3.1. Validation of the methodology for various multilayer test cases (black crosses = measurements, blue line = model output, red lines = soil model parameters)

It should be noted that while the agreement between calculated and expected values of the apparent resistivity is excellent (blue curve goes through the data points), the RMS error $\sqrt{N^{-1}\Psi}$ and the errors in the soil parameters, in particular in the deeper ones, tends to grow with the number of layers.

It is clear that with a high number of layers, i.e. a higher number of degrees of freedom, some parameters cannot be identified with high accuracy.In all cases, the validations have been performed using the same exit condition for the trust region method and the same precision for the calculations. It is noteworthy that it is not easy (or not possible) to investigate what happens in the deeper layers, in particular below the interface between layers with a high reflection coefficient (i.e. low transmission coefficient). One of the causes of this behavior is the presence of round off errors in the numerical calculations. In these artificial validation cases, as anticipated, (11) has a minimum with zero value. The trust region method is able to find a solution with small objective function values in all cases and almost the exact solution is found when the number of layers is limited to 2 or 3. The RMS error of all models in Table III is anyway very low (< 0.01 %) and this means that the behavior of the calculated model corresponds very well to the true model. The proposed approach has been tested also with very high number of layers: the parameters in Table IV (see Fig. 3.2) have been calculated with a ten layers model by applying weights to the measured values with smaller and bigger electrodes spacing. Errors in the central layers are higher than those in the upper and lower layers. One notices that soil parameters may present significant errors although the expected and measured resistivities match well.

TABLE IV
TEN LAYERS SOIL MODEL PARAMETERS EXPECTED AND CALCULATED

	Exp.	Calc.		Exp.	Calc.
ρ_1	100.0	99.98 (-0.020%)	h_1	2.000	2.000
ρ_2	50.00	49.88 (-0.24%)	h_2	5.000	4.990 (-0.20%)
ρ_3	200.0	202.1 (+1.0%)	h_3	10.00	9.250 (-7.5%)
ρ_4	50.0	63.52 (+27%)	h_4	10.00	12.16 (+22%)
ρ_5	300.0	186.1 (-38%)	h_5	10.00	14.70 (+47%)
ρ_6	100.0	201.7 (+102%)	h_6	10.00	11.71 (+12%)
ρ_7	300.0	162.0 (-46%)	h_7	10.00	14.39 (+44%)
ρ_8	100.0	159.9 (+60%)	h_8	10.00	13.30 (+33%)
ρ_9	300.0	148.1 (-51%)	h_9	10.00	16.14 (+61%)
ρ_{10}	100.0	99.99 (-0.010%)	RMS %	-	0.01481

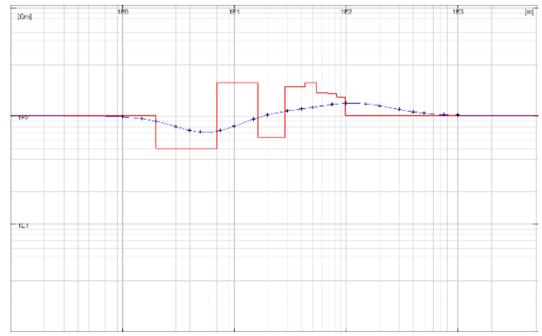


Fig. 3.2. Soil resistivity measured values with a ten layers soil model

VI. A CASE STUDY

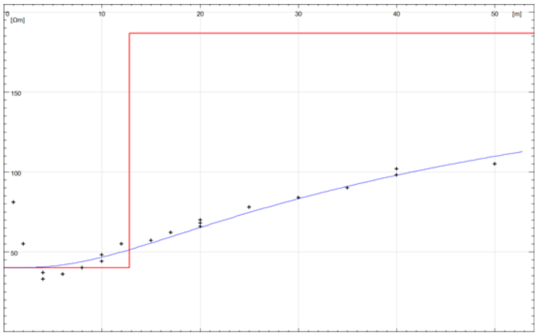
Let's consider a set of measured apparent soil resistivity values, shown in Table V and obtained with Wenner's method and rounded to the nearest integer. Note that some measurements are obtained with the same value of the spacing *a* as is typical in the engineering practice, which precludes the minimum of the objective function to be zero.

TABLE V
MEASURED APPARENT SOIL RESISTIVITY AS A FUNCTION OF THE ELECTRODES SPACING

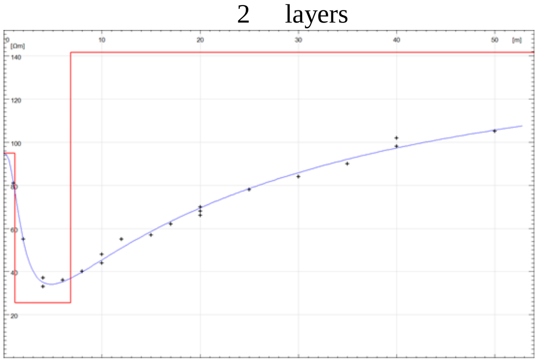
a	ρ	a	ρ
1.000	81.00	17.00	62.00
2.000	55.00	20.00	66.00
4.000	33.00	20.00	68.00
4.000	37.00	20.00	70.00
6.000	36.00	25.00	78.00
8.000	40.00	30.00	84.00
10.00	44.00	35.00	90.00
10.00	48.00	40.00	98.00
12.00	55.00	40.00	102.0
15.00	57.00	50.00	105.0

TABLE VI
SOIL MODEL PARAMETERS CALCULATED BY SRA

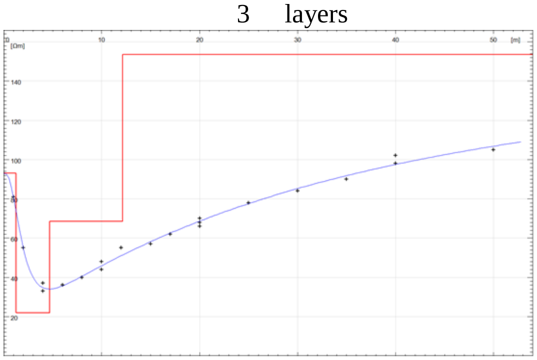
	2 layers	3 layers	4 layers	5 layers
ρ_1	40.06	94.83	93.06	93.08
ρ_2	186.6	25.48	21.86	22.17
ρ_3	-	141.7	68.52	85.27
ρ_4	-	-	153.5	299.2
ρ_5	-	-	-	3.300
h_1	12.81	1.167	1.259	1.254
h_2	-	5.669	3.440	3.786
h_3	-	-	7.460	16.13
h_4	-	-	-	39.98
RMS %	15.10	3.497	3.261	3.217



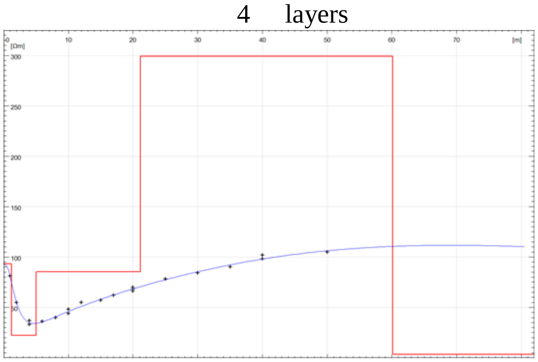
2 layers



3 layers



4 layers



5 layers

Fig. 4.1. Soil resistivity measured values multilayers soil models (black crosses = measurements, blue line = objective function, red lines = soil model parameters)

Using this data, the soil parameters obtained by the trust region optimizer for the case of 2 to 5 layer models are those shown in Table VI (see also Fig. 4.1).

The double layer soil model is clearly not appropriate since a double layer soil model can well approximate only a set of monotonically increasing or decreasing measurements, which is not the case for the present dataset.

A triple layer soil model gives a low RMS error with a smallest possible number of layers and is probably the most appropriate model for the specific case since indeed such a model can well approximate a set of measurements with a minimum or a maximum. A problem expert could understand that the more appropriate model in this case is a triple layer soil model by simply observing the distribution of the measured values.

Models with a higher number of layers give a slightly lower RMS error but with a higher number of layers and therefore require a more complex calculation.

The use of the RMS error as criterion to evaluate the quality of a model is good but the best criterion of judgment requires the visual comparison of model and measures.

For instance, an expert user of this methodology can modify results by adding artificial measurements or removing doubtful measurements or adding constraints in the used optimization method.

In real cases, the model should reflect the geological reality, if known. In the specific case, if the geological observations indicate four distinct layers, the 4-layer model is probably the most suitable.

If the geological observations indicate a specific range of resistivity for each layer, these information should be used by introducing suitable constraints in the optimization algorithm.

The choice of the right number of layers and the introduction of constraints requires a preliminary knowledge of soil stratifications and properties and this is not usual. In practical cases, if no additional information is available, the number of layers should be set as the minimum number over which the RMS error does not decrease significantly.

Moreover, it is evident that variations in soil resistivity at great depth cannot be detected with good accuracy from limited measurements obtained on the soil surface, thus thin layers at great depth should be avoided in soil modeling.

VII. Conclusion

This paper concerns the soil modeling for grounding systems analysis. As known, the most suitable model for grounding system analysis is the multilayer soil model with a number of layers depending on the specific case but in practical cases between three and five.

The choice of the number of layers and the introduction of constraints consistent with the geological reality can aid in the model parameter identification.

From the mathematical point of view, such problem is ill-posed and can be solved only in an approximate sense.

The paper describes a methodology implemented in the SRA module of the XGSLab® simulation environment. The SRA module can obtain the parameters of a multilayer soil model with an arbitrary number of layers, starting from the apparent soil resistivity values measured with Wenner's and/or Schlumberger's method.

The paper also shows some validations in case of a multilayer soil model with 2, 3, 4 and 5 layers and shows that with a high number of layers the parameters of the deeper ones cannot be evaluated with high accuracy. As intuitive, it is not easy to identify what happens at great depth starting from limited measurements carried out on the soil surface. In particular, it is not easy or not possible to evaluate with high accuracy the parameters (resistivity and thickness) of layers in deeper soil, in particular below the interface between layers with a high reflection coefficient.

Anyway, in all cases the method is able to obtain soil parameters which corresponds to a model with a low RMS error, and therefore well representative of the measurements.

The paper finally shows a case study based on real measurements and discusses some practical tips about the choice of the number of layers.

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