SINT Srl

ONE STEP AHEAD

XGSLab™

THE STATE OF THE ART OF THE
ELECTROMAGNETIC SIMULATION FOR
POWER, GROUNDING AND
LIGHTNING PROTECTION SYSTEMS

XGSLab™ Vs. CDEGS®
SOIL RESISTIVITY ANALYSIS

[October 2019]

Introduction

The program XGSLab includes the following module:

- SRA (Soil Resistivity Analyzer)

The program CDEGS® (developed by SES & Technologies ltd. Canada) includes many modules and in particular:

- RESAP®

The two modules are based on similar assumptions but use different optimization algorithm.

SRA calculates the parameters (low frequency soil resistivity and thickness for each layer) that best fit the measured data using an optimization procedure that finds the minimum of the following squared error function taking into account constrains:

$$\psi(\rho_1,\rho_2,\cdots\rho_n,h_1,h_2,\cdots h_{n-1})=\sum_{i=1}^{N} w_i r_i\left[\frac{\rho_m(a_i,c_i)-\rho_c(a_i,c_i,\rho_1,\rho_2,\cdots\rho_n,h_1,h_2,\cdots h_{n-1})}{\rho_m(a_i,c_i)}\right]^2$$

$$w_i = 1+\frac{10}{N}\left|2i-N-1\right|$$

Where (a = with, b = without average of the measures with the same electrode spacing respectively):

- Ψ = squared error function
- w_i = weight function (optional)
- r_i = a) 1, b) number of occurrences
- $\rho_m(a_i,c_i)$ (Ωm) = a) measured apparent soil resistivity b) aggregate apparent soil resistivity
- $\rho_c(a_i,c_i)$ (Ωm) = calculated apparent soil resistivity
- a_i, c_i (m) = electrodes spacing
- N = a) number of measures, b) number of aggregate measures

About the parameter r_i and N :

a) if the flag “Average” is selected, SRA groups the measures with the same pair "a" and "c" by making the average and note the number of occurrences
b) if the flag “Average” is not selected, SRA uses all measures and the number of occurrences is 1

Several methods to calculate the multilayer model parameters have been proposed in literature.
Different methods are based on different optimization algorithm and can be classified as:

- direct search method: downhill simplex method (DSM), genetic algorithms (GA)
- gradient based methods: steepest descent method (SDM), Levenberg Marquardt method (LMM), conjugate gradient method (CGM), trust region method (TRM)

SRA uses TRM while RESAP® uses LMM.

The TRM is one of the most important and recent (1982) numerical optimization methods in solving nonlinear programming problems and is used in many fields (engineering, economic, military …). In comparison with others algorithms, TRM is reliable and robust, can be applied to ill-conditioned problems and has very strong convergence properties. This justify in many case the best results obtained with SRA in comparison with RESAP®.

SRA can includes constraints in results (minimum and maximum values) in order to avoid unrealistic results, averaging in order to allows to consider very large measures set and weigh functions in order to improve results with small and large electrodes spacing.

In the following, first some validations of SRA in ideal cases and then, a comparison between SRA and RESAP® in real cases.

SRA VALIDATION

In order to validate SRA in cases of a two, three, four and five layers soil model, some perfect multilayers soil model with the parameters in Table 1 has been considered.

	Two Layers	Three Layers	Four Layers	Five Layers
ρ_1 (Ωm)	100.0	100.0	100.0	100.0
ρ_2 (Ωm)	50.00	50.00	50.00	50.00
ρ_3 (Ωm)	-	200.0	200.0	200.0
ρ_4 (Ωm)	-	-	75.00	20.00
ρ_5 (Ωm)	-	-	-	300.0
h_1 (m)	2.000	2.000	2.000	2.000
h_2 (m)	-	6.000	6.000	6.000
h_3 (m)	-	-	15.00	10.00
h_4 (m)	-	-	-	15.00

Table 1: Multilayer soil model parameters used for validations

The corresponding apparent soil resistivities ρ_E values measured with the Wenner method as a function of the electrodes spacing can be calculated with the following formula:

$$\rho_E(a) = \rho_1 + 4\rho_1 a \int_0^\infty B_n(\lambda)\left[J_0(\lambda a) - J_0(2\lambda a)\right] d\lambda$$

where

- ρ_1 (Ωm) = upper layer soil resistivity
- a (m) = electrodes spacing
- $B_n(\lambda)$ = Kernel function for a n-layers soil model (function of soil parameters)
- $J_0(\lambda a)$ = Bessel function (first kind, zero order)

Using the Wenner method, the measured Wenner resistance R_W values should be:

$$R_W(a) = \frac{\rho_E(a)}{2\,\pi\,a}$$

By entering in SRA directly the apparent soil resistivity or alternatively the Wenner resistance values, the initial soil model parameters should be obtained.

The measured apparent soil resistivity as a function of the electrodes spacing value calculated with the above formula are indicated in Table 2.

a (m)	Two Layers ρ_E (Ωm)	Three Layers ρ_E (Ωm)	Four Layers ρ_E (Ωm)	Five Layers ρ_E (Ωm)
1.000	97.59	97.65	97.65	97.93
1.500	93.54	93.74	93.74	93.89
2.000	88.27	88.75	88.73	88.80
3.000	77.50	79.00	78.95	78.88
4.000	69.01	72.27	72.15	71.91
5.000	63.17	68.89	68.65	68.16
7.500	55.83	70.00	69.23	67.70
10.00	53.08	77.24	75.49	72.26
15.00	51.26	94.69	89.56	81.36
20.00	50.69	110.0	99.75	85.75
30.00	50.30	132.6	108.7	85.53
40.00	50.17	147.8	109.0	82.88
50.00	50.11	158.5	105.8	82.32
75.00	50.05	174.5	95.58	91.03
100.0	50.03	182.9	88.13	106.1

Table 2: Measured apparent soil resistivity as a function of the electrodes spacing

With the values in Table 2 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: Off
- Initial conditions: Default
- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA are as indicated in Table 3.

	Two Layers	Three Layers	Four Layers	Five Layers
ρ_1 (Ωm)	100.0	100.0	100.0	100.3
ρ_2 (Ωm)	50.00	50.01	50.03	50.37
ρ_3 (Ωm)	-	200.0	200.7	223.3
ρ_4 (Ωm)	-	-	75.07	26.11
ρ_5 (Ωm)	-	-	-	302.4
h_1 (m)	2.000	1.999	1.999	1.981
h_2 (m)	-	6.004	6.012	6.217
h_3 (m)	-	-	14.89	8.514
h_4 (m)	-	-	-	19.96
RMS Error %	0.00289	0.00748	0.00580	0.00897

Table 3: Soil model parameters calculated by SRA and related RMS error

In all cases, the agreement between calculated and expected values is excellent (see also following figures). The RMS Error is always negligible or very low.

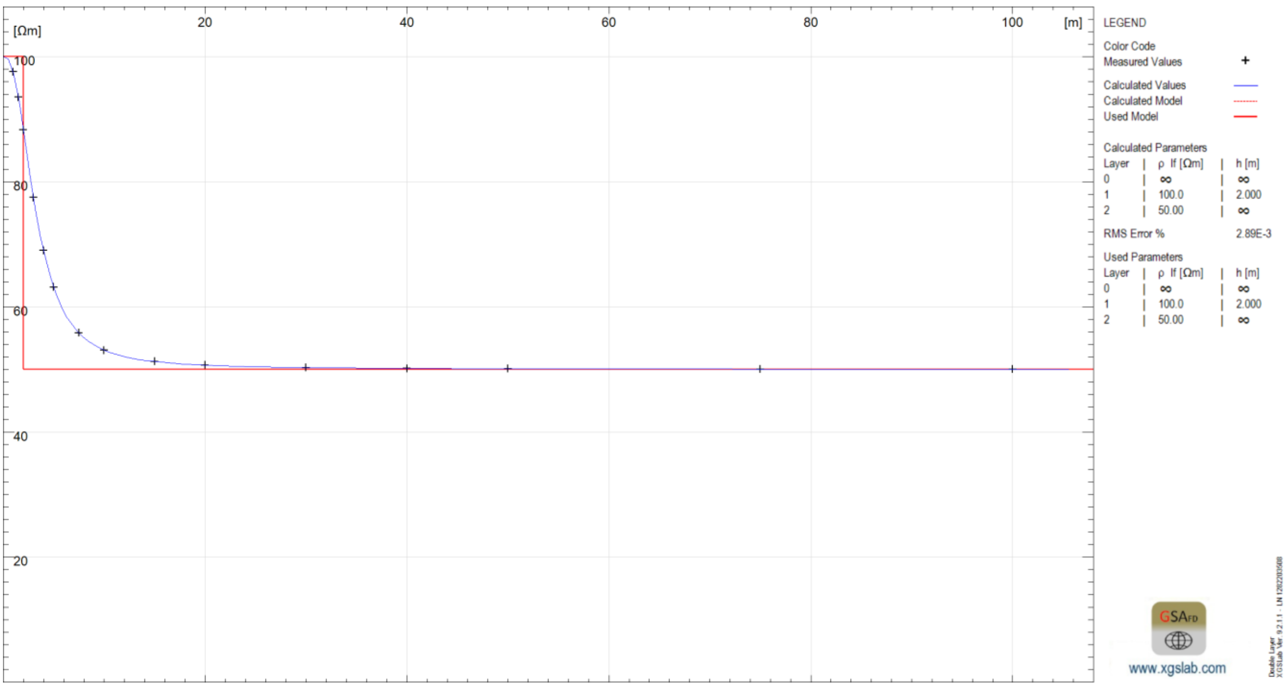

Figure 1 Soil resistivity measured values and double layers soil model

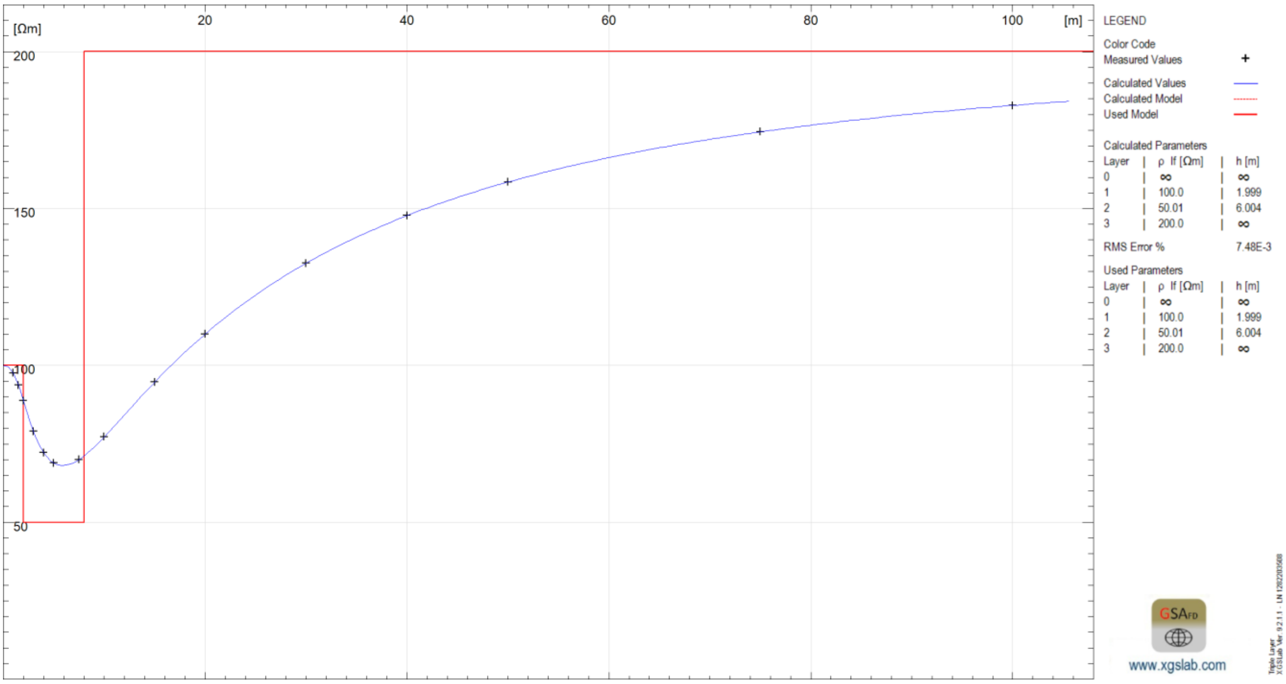

Figure 2: Soil resistivity measured values and triple layers soil model

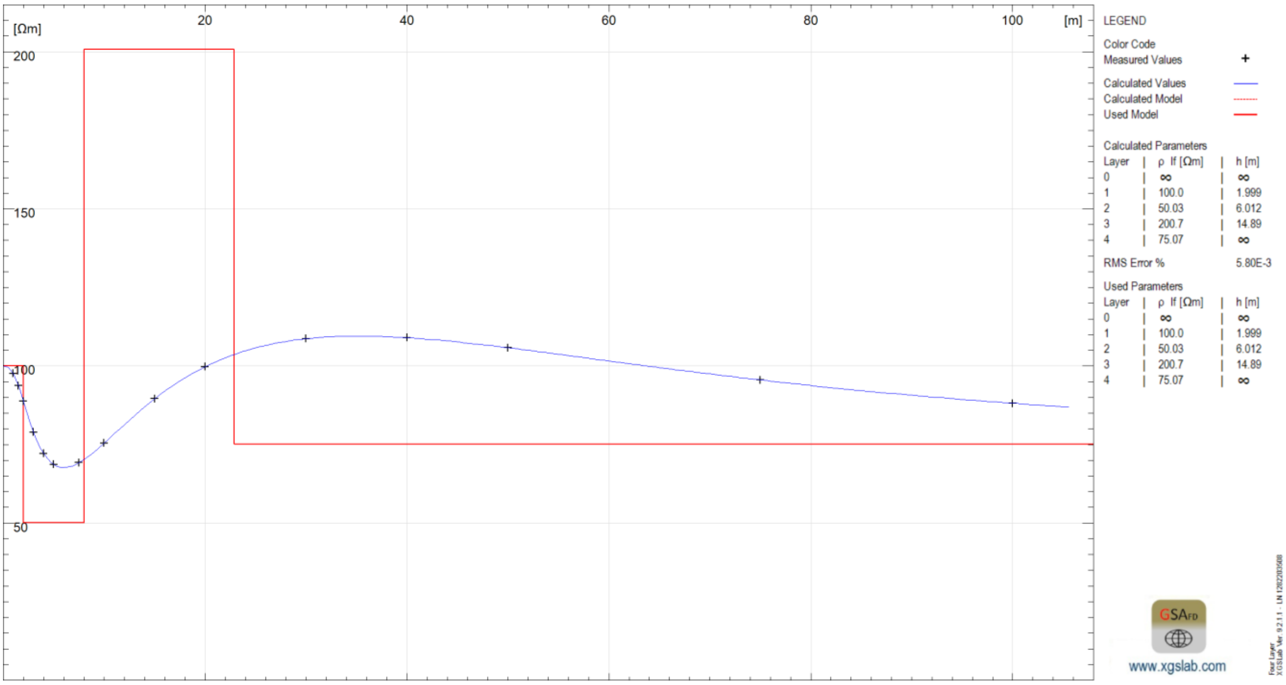

Figure 3: Soil resistivity measured values and four layers soil model

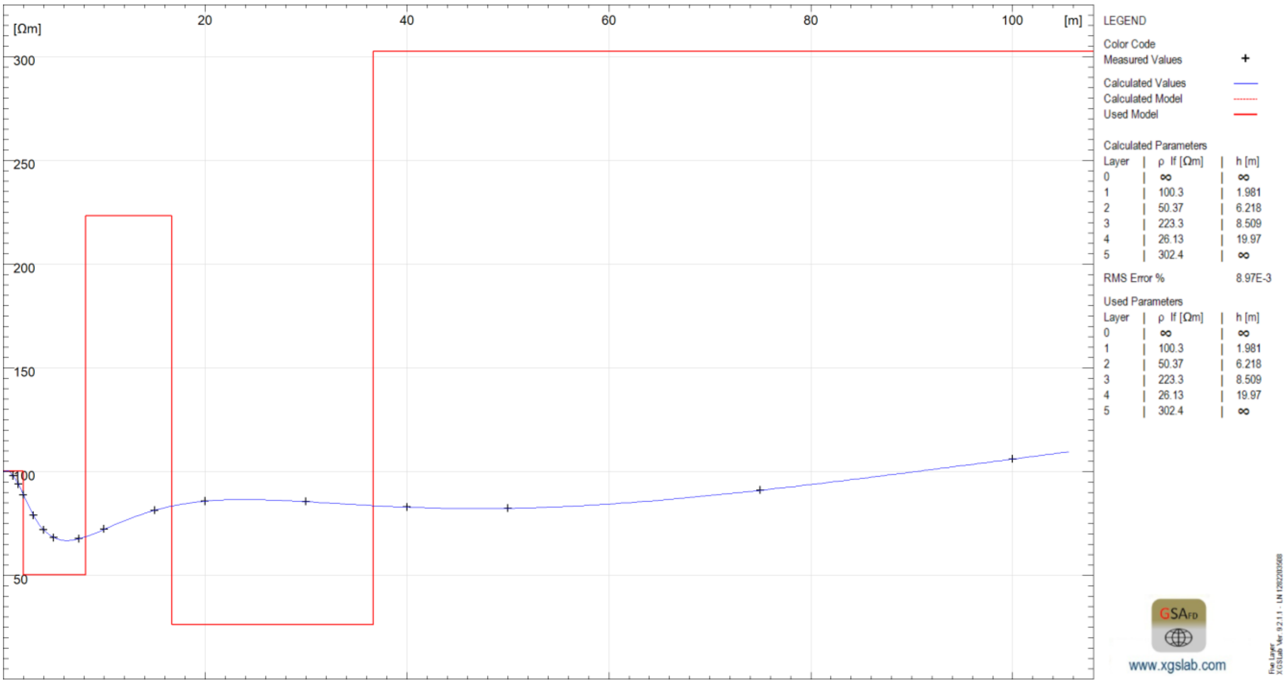

Figure 4: Soil resistivity measured values and five layers soil model

Comparison between SRA and RESAP®

Case 1 - Triple layer soil model

The measured apparent soil resistivity and correspondent Wenner resistance as a function of the electrodes spacing value are indicated in Table 4.

$a = c$ (m)	ρ_E (Ω m)	R_W (Ω)
1.000	346.8	55.19
1.500	507.1	53.80
2.000	431.0	34.30
3.000	317.0	16.82
4.500	167.1	5.910
6.000	116.5	3.090
9.000	71.30	1.261
13.50	51.70	0.6095
18.00	49.80	0.4403
27.00	54.30	0.3201
36.00	72.40	0.3201
54.00	101.8	0.3000

Table 4: Measured apparent soil resistivity and resistance as a function of the electrodes spacing

With the values in Table 4 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: On
- Initial conditions: Default
- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA and brackets from RESAP® are (see Figure 5):

- Soil resistivity of the upper layer = 420.4 (435.1) Ωm
- Soil resistivity of the central layer = 39.63 (42.28) Ωm
- Soil resistivity of the bottom layer = 509.0 (267.21) Ωm
- Upper layer thickness = 2.784 (2.668) m
- Central layer thickness = 24.87 (23.22) m
- RMS Error = 11.48% (11.61%)

The agreement between calculated and expected values is good except the soil resistivity of the bottom layer. In this case the resistivity of the bottom layer is the one given by the constraints. It is possible to reduce the RMS Error (up to 11.27%) by increasing the bottom layer resistivity constraint. In this case the differences with RESAP® are more evident but the RMS Error improves.

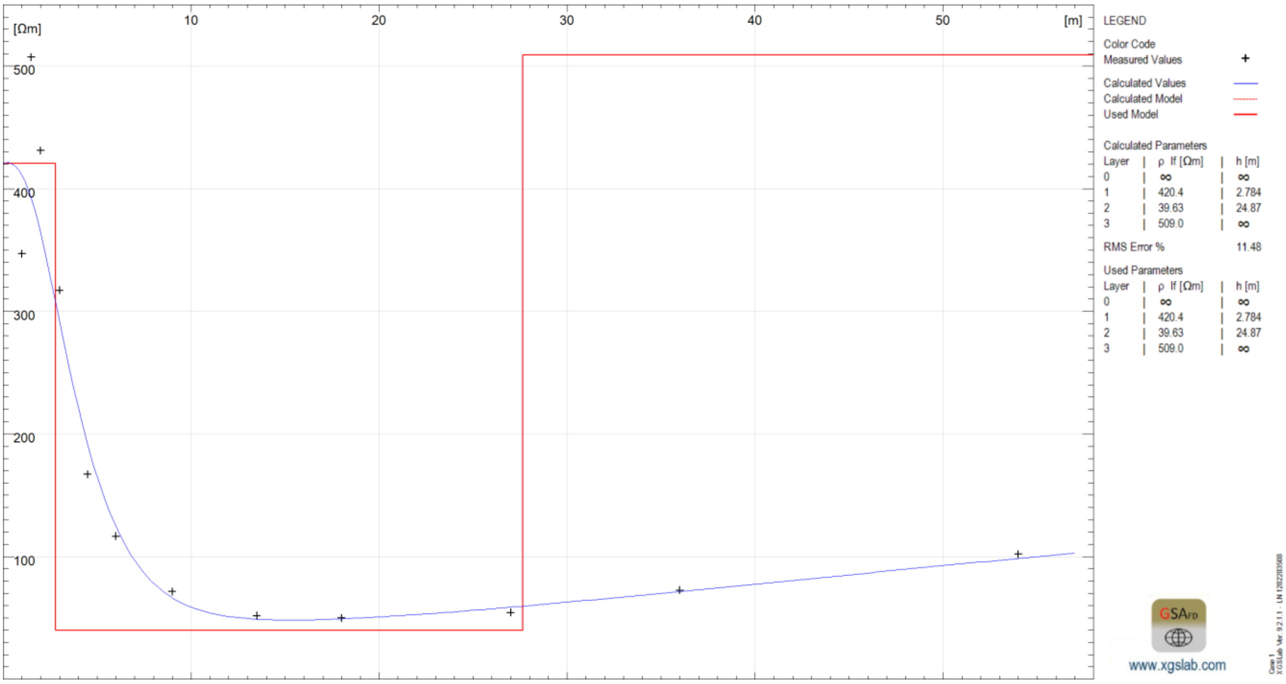

Figure 5: Soil resistivity measured values and triple layers soil model

The same measures set has been tested with a 4 and a 5 layers soil model. In both cases the RMS Error is lower than in the 3 layers soil model calculation (11.20% and 10.90% versus 11.48%).

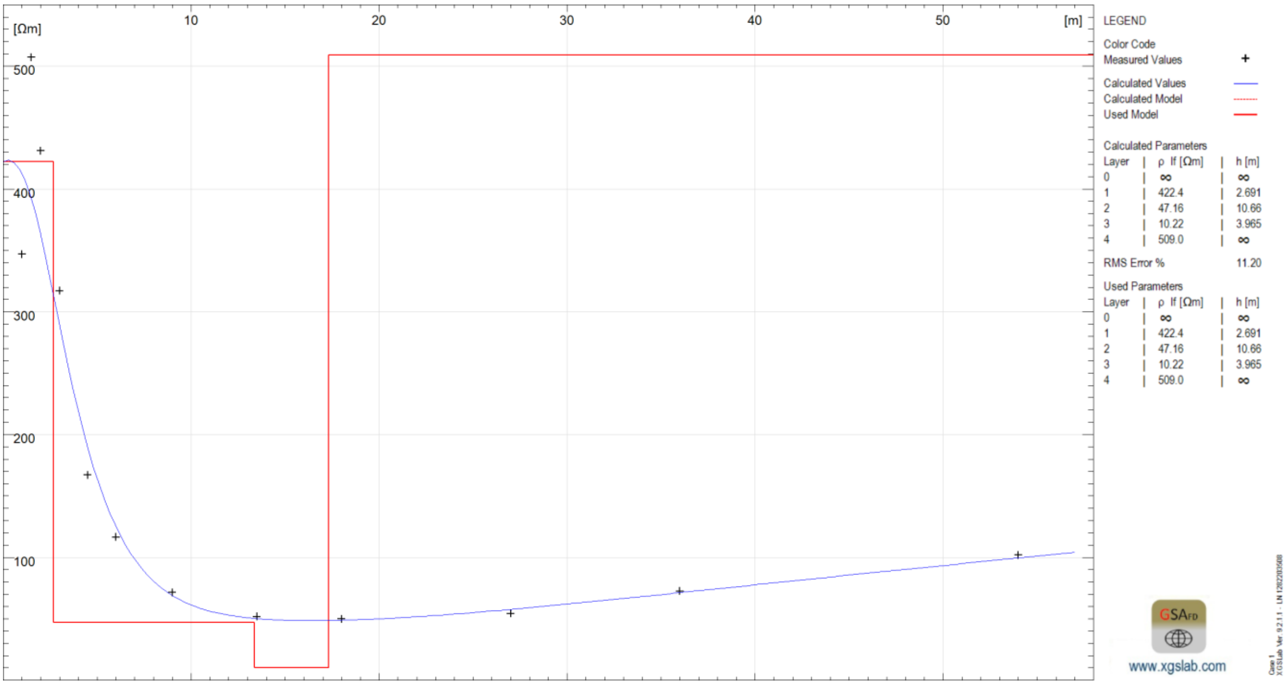

Figure 6: Soil resistivity measured values and four layers soil model – Linear scale

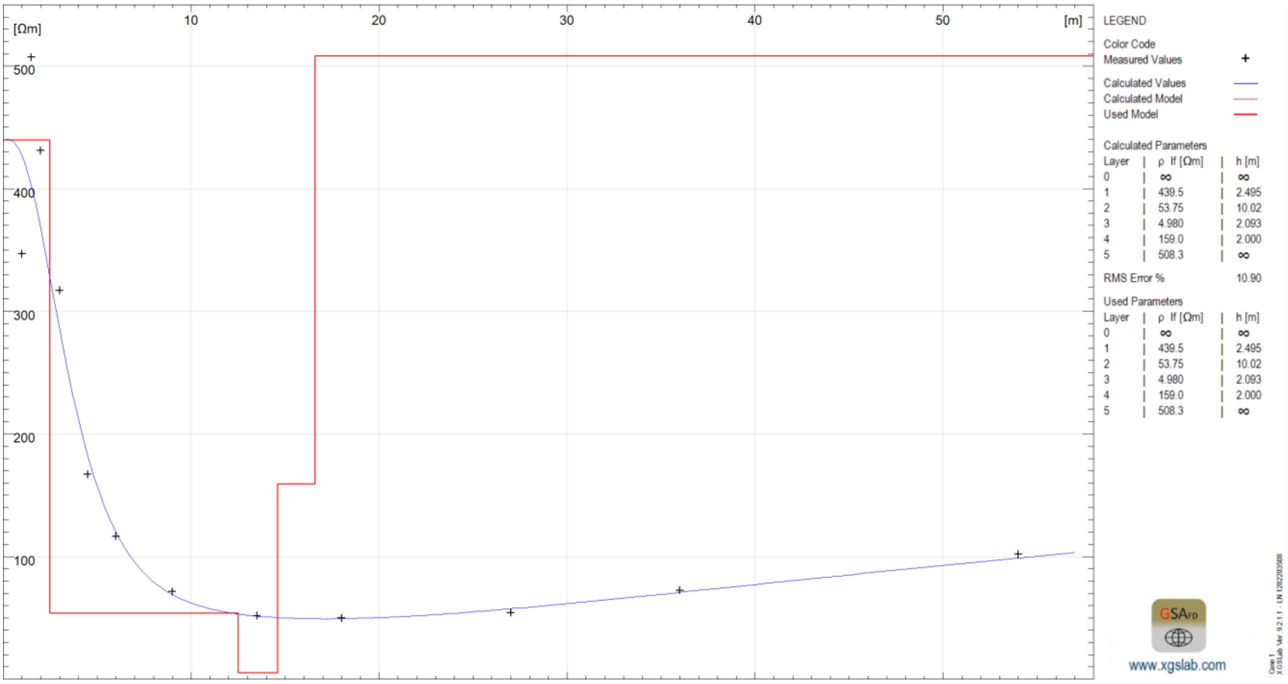

Figure 7: Soil resistivity measured values and five layers soil model – Linear scale

<u>Case 2 - Triple layer soil model</u>

The measured apparent soil resistivity and correspondent Wenner resistance as a function of the electrodes spacing value are indicated in Table 5.

$a = c$ (m)	ρ_E (Ω m)	R_W (Ω)
1.000	80.20	12.76
1.500	58.60	6.218
2.000	41.00	3.263
3.000	35.10	1.862
4.500	36.20	1.280
6.000	36.60	0.9708
9.000	44.10	0.7799
13.50	54.30	0.6402
18.00	66.70	0.5898
27.00	79.70	0.4698
36.00	95.00	0.4200
54.00	101.8	0.3000

Table 5: Measured apparent soil resistivity and resistance as a function of the electrodes spacing

With the values in Table 5 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: On
- Initial conditions: Default

- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA and brackets from RESAP® are (see Figure 8):

- Soil resistivity of the upper layer = 133.0 (120.2) Ωm
- Soil resistivity of the central layer = 29.62 (28.57) Ωm
- Soil resistivity of the bottom layer = 128.9 (126.9) Ωm
- Upper layer thickness = 0.6804 (0.7399) m
- Central layer thickness = 6.359 (5.948) m
- RMS Error = 3.073% (3.421%)

The agreement between calculated and expected values is good.

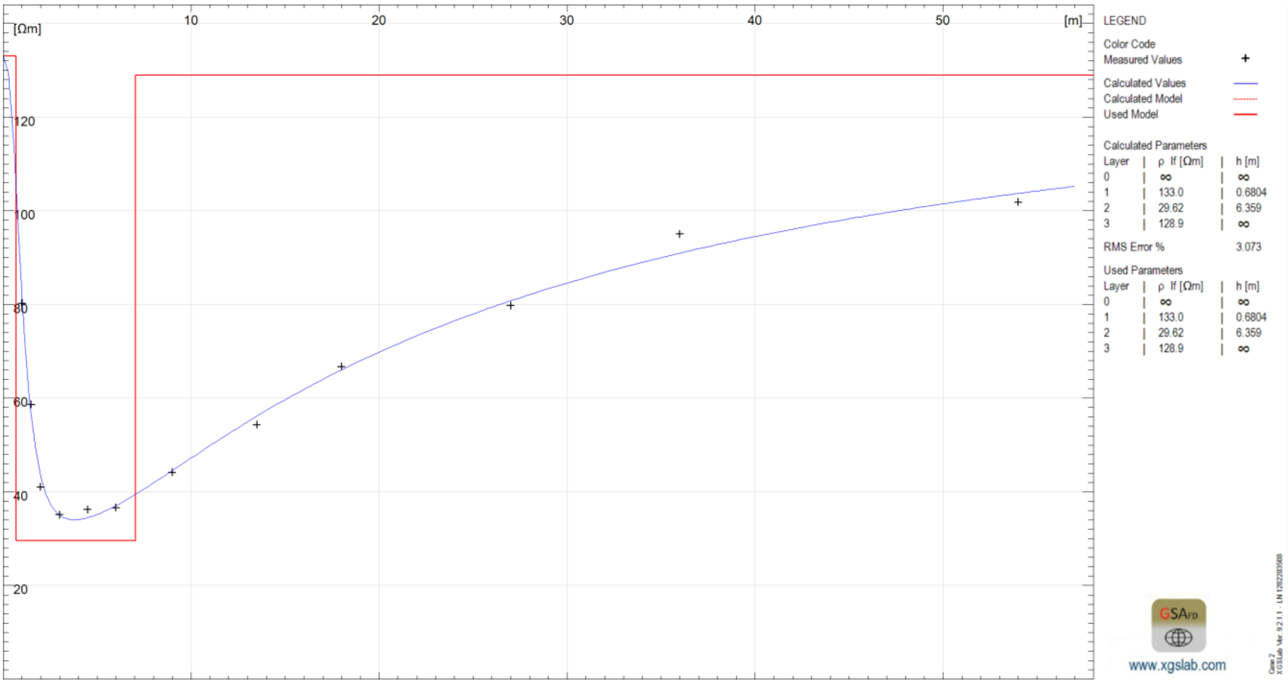

Figure 8: Soil resistivity measured values and triple layers soil model

The RMS Error is quite low and is not necessary to look for a better result with more layers.

<u>Case 3 - Triple layer soil model</u>

The measured apparent soil resistivity and correspondent Wenner resistance as a function of the electrodes spacing value are indicated in Table 6.

$a = c$ (m)	ρ_E (Ω m)	R_W (Ω)
1.000	333.0	53.00
1.500	237.5	25.20
2.000	140.2	11.16
3.000	112.2	5.952
4.500	91.00	3.218
6.000	116.5	3.090
9.000	52.00	0.9196
13.50	41.60	0.4904
18.00	43.00	0.3802
27.00	49.20	0.2900
36.00	56.50	0.2498
54.00	57.70	0.1701

Table 6: Measured apparent soil resistivity and resistance as a function of the electrodes spacing and depth

With the values in Table 6 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: On
- Initial conditions: Default
- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA and brackets from RESAP® are (see Figure 9 and Figure 10):

- Soil resistivity of the upper layer = 676.1 (428.5) Ωm
- Soil resistivity of the central layer = 124.1 (109.0) Ωm
- Soil resistivity of the bottom layer = 49.24 (45.64) Ωm
- Upper layer thickness = 0.5910 (0.7750) m
- Central layer thickness = 2.354 (2.885) m
- RMS Error = 16.68% (16.85%)

The agreement between calculated and expected values is good.

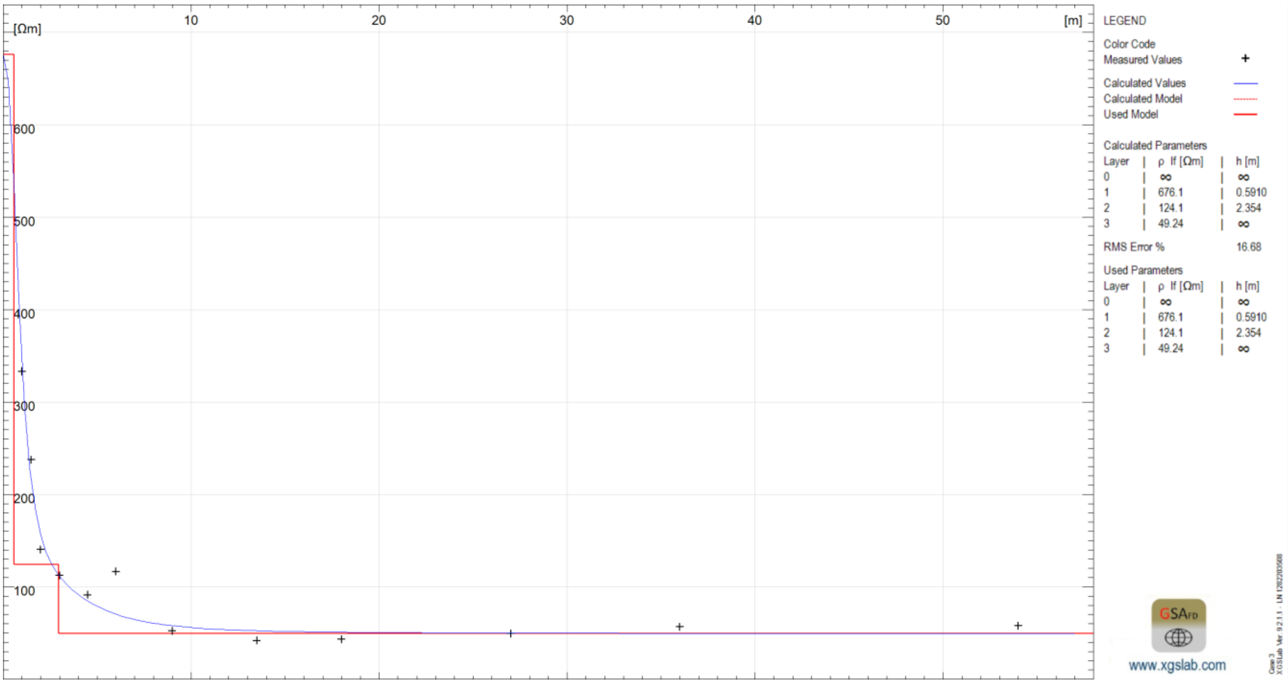

Figure 9: Soil resistivity measured values and triple layers soil model – Linear scale

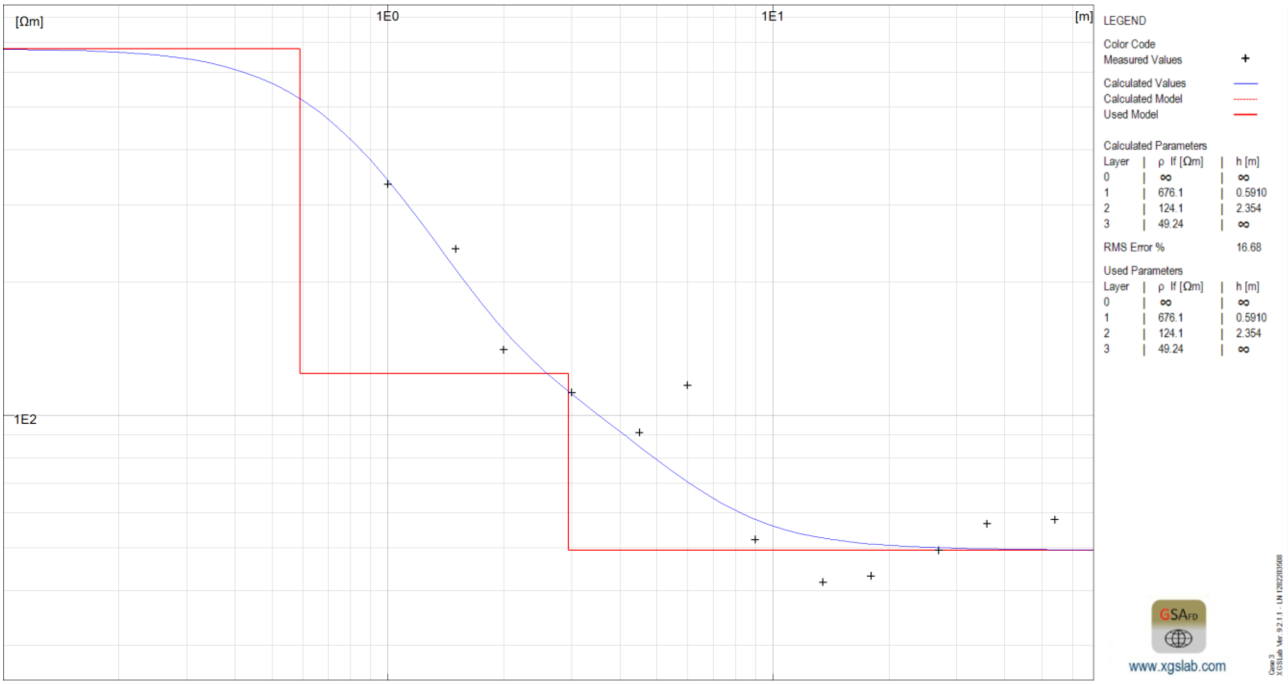

Figure 10: Soil resistivity measured values and four triple soil model – Logarithmic scale

The same measures set has been tested with a 4 and a 5 layers soil model. In both cases the RMS Error is far lower than in the 3 layers soil model calculation (10.89% and 11.72% versus 16.68%).
The difference between 4 and 5 layers is not substantial.

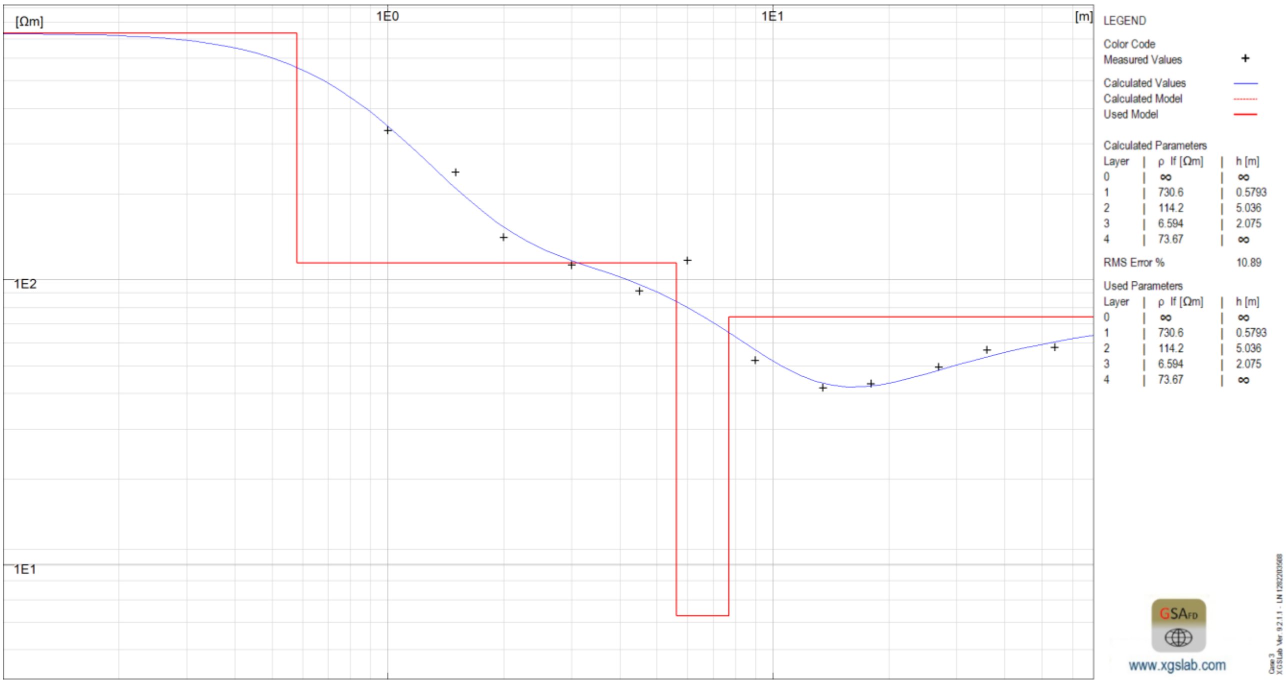

Figure 11: Soil resistivity measured values and four layers soil model – Logarithmic scale

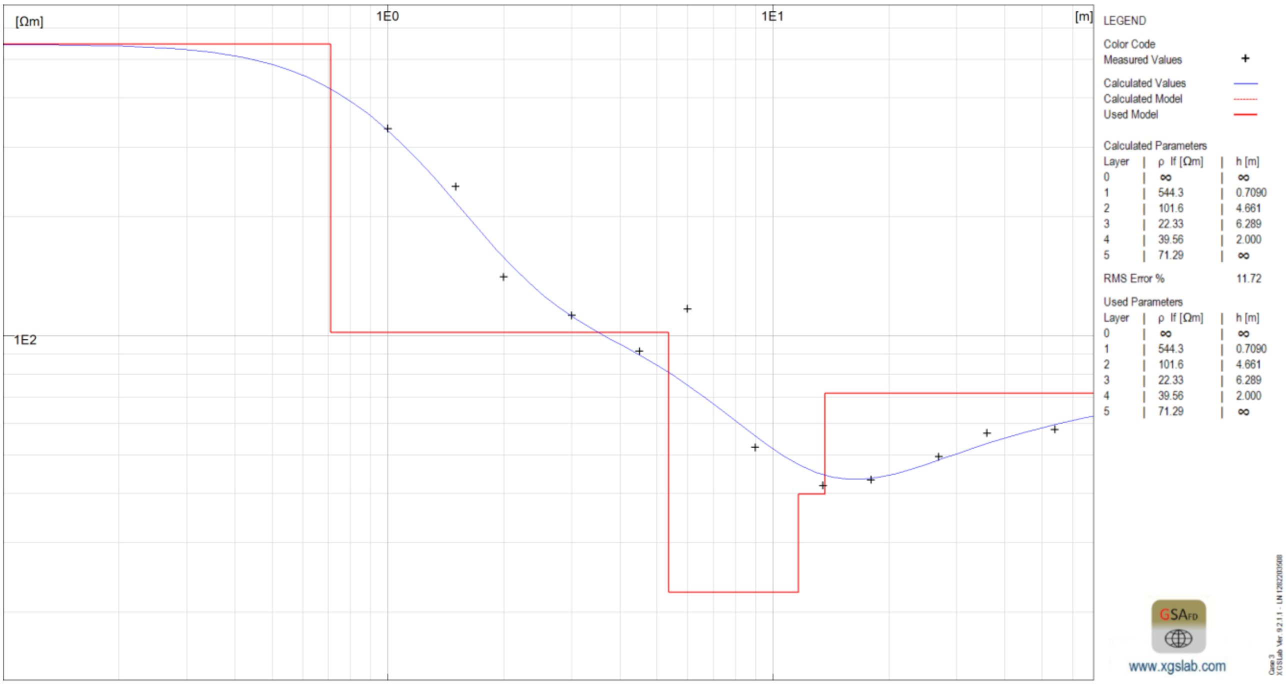

Figure 12: Soil resistivity measured values and five layers soil model – Logarithmic scale

Case 4 - Triple layer soil model

The measured apparent soil resistivity and correspondent Wenner resistance as a function of the electrodes spacing value are indicated in Table 7.

$a=c$ (m)	ρ_E (Ω m)	R_W (Ω)
1.000	321.7	51.20
1.500	383.6	40.70
2.000	454.9	36.20
3.000	514.6	27.30
4.500	571.1	20.20
6.000	501.4	13.30
9.000	369.8	6.539
13.50	242.6	2.860
18.00	131.2	1.160
27.00	71.30	0.4203
36.00	63.30	0.2798
54.00	61.10	0.1801

Table 7: Measured apparent soil resistivity and resistance as a function of the electrodes spacing

With the values in Table 7 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: On
- Initial conditions: Default
- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA and brackets from RESAP® are (see Figure 13 and Figure 14):

- Soil resistivity of the upper layer = 271.3 (250.1) Ωm
- Soil resistivity of the central layer = 1869 (958.0) Ωm
- Soil resistivity of the bottom layer = 58.17 (57.18) Ωm
- Upper layer thickness = 1.245 (0.9220) m
- Central layer thickness = 1.895 (3.813) m
- RMS Error = 3.834% (3.724%)

The agreement between calculated and expected values is good.

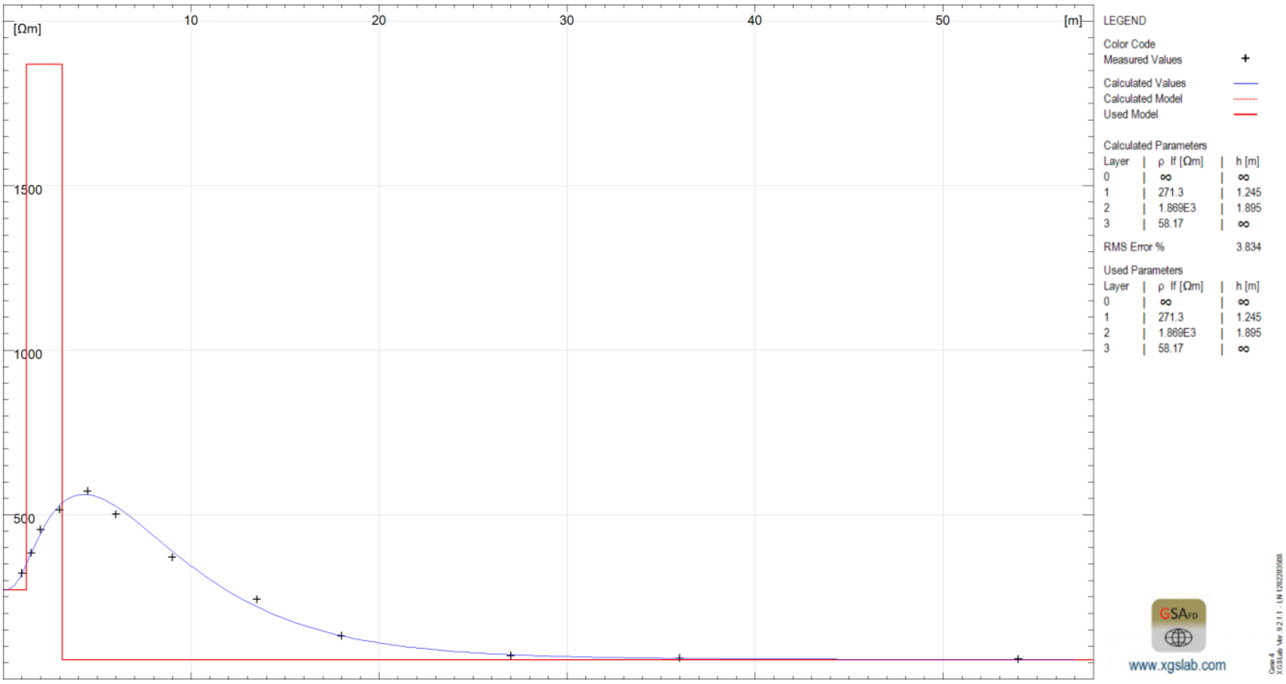

Figure 13: Soil resistivity measured values and triple layers soil model – Linear scale

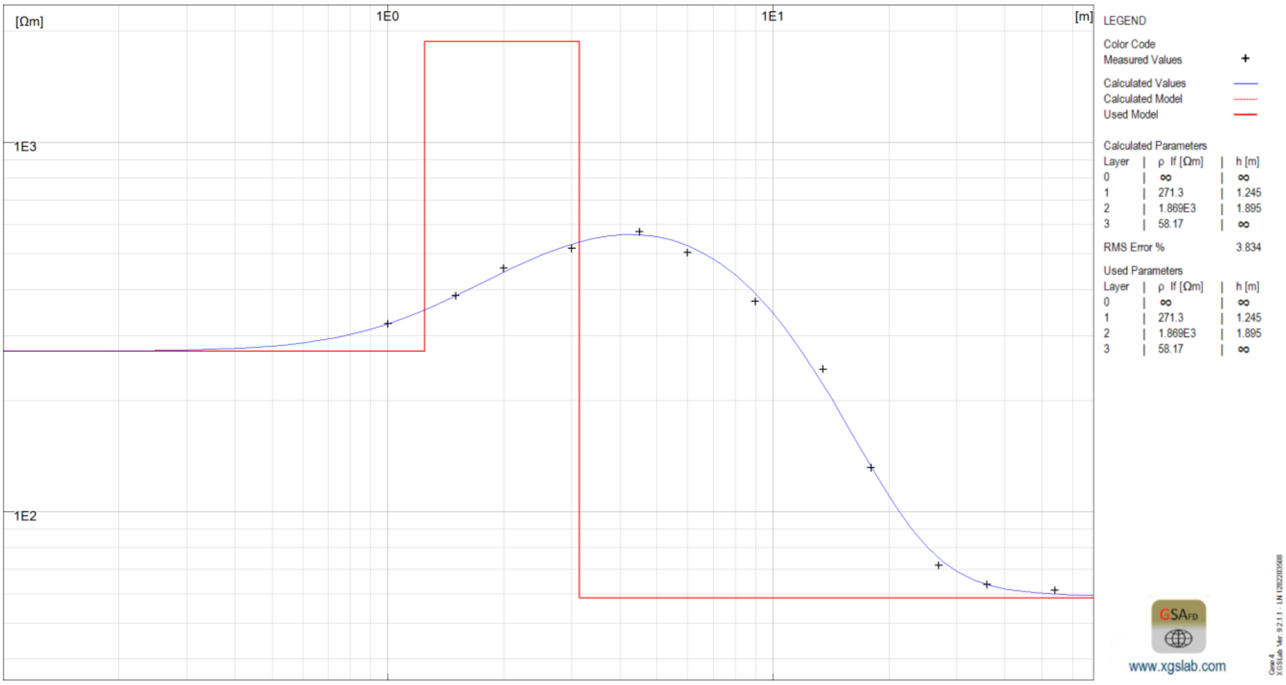

Figure 14: Soil resistivity measured values and triple layers soil model – Logarithmic scale

The RMS Error is quite low and is not necessary to look for a better result with more layers.

In this case we have also tried to calculate the soil model without using the weights (following picture).

The resulting RMS is (as expected) better than with constraints.

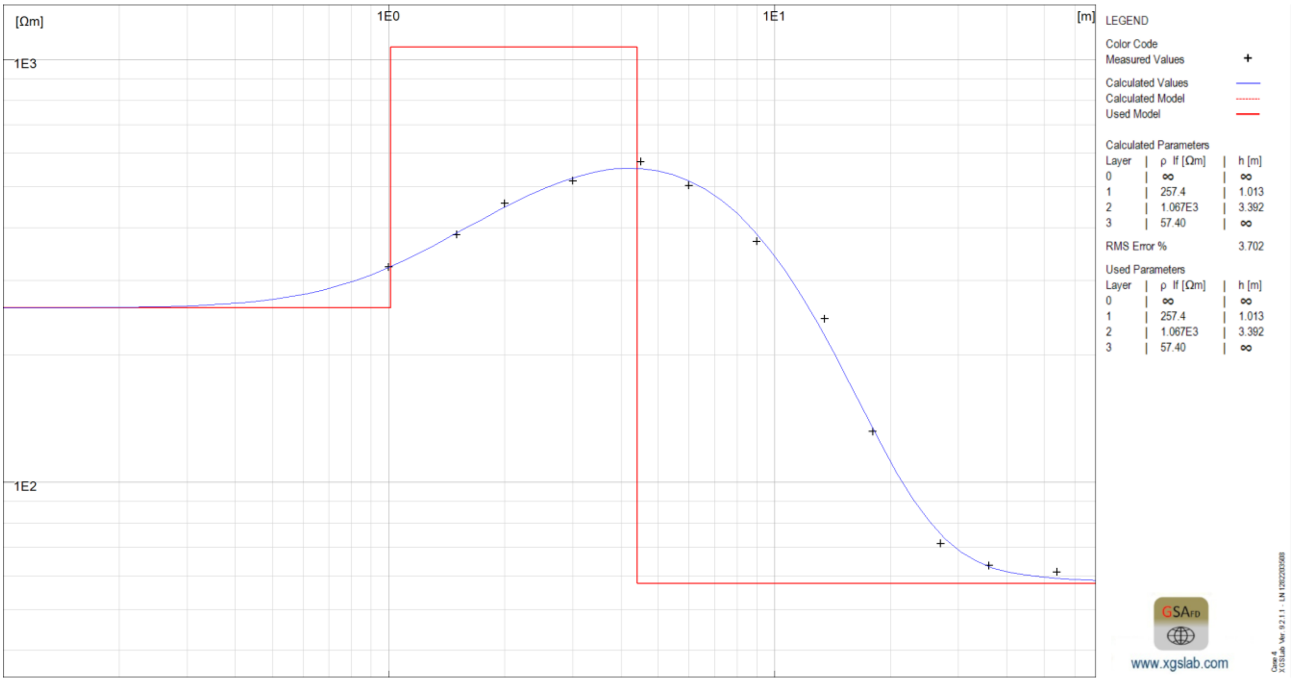

Figure 15: Soil resistivity measured values and triple layers soil model – Logarithmic scale

<u>Case 5 – Four layers soil model</u>

The measured apparent soil resistivity and correspondent Wenner resistance as a function of the electrodes spacing value are indicated in Table 8.

$a = c$ (m)	ρ_E (Ω m)	R_W (Ω)
1.000	283.4	45.10
1.500	260.1	27.60
2.000	201.3	16.02
3.000	170.0	9.019
4.500	147.9	5.231
6.000	138.4	3.671
9.000	154.9	2.739
13.50	173.0	2.040
18.00	187.7	1.660
27.00	173.0	1.020
36.00	181.0	0.8002
54.00	162.9	0.4801

Table 8: Measured apparent soil resistivity and resistance as a function of the electrodes spacing

With the values in Table 8 and the following calculation options:

- Average: On or Off (there are no recurrences)
- Weights: On
- Initial conditions: Default
- Lower constrains: Default
- Upper constrains: Default

the parameters calculated by SRA and brackets from RESAP® are (see Figure 16):

- Soil resistivity of the upper layer = 323.6 (307.4) Ωm
- Soil resistivity of the second layer = 117.0 (87.35) Ωm
- Soil resistivity of the third layer = 743.4 (254.9) Ωm
- Soil resistivity of the bottom layer = 148.2 (152.7) Ωm
- Upper layer thickness = 1.195 (1.467) m
- Second layer thickness = 6.416 (3.014) m
- Third layer thickness = 2.413 (9.093) m
- RMS Error = 3.174% (3.435%)

The agreement between calculated and expected values is good.

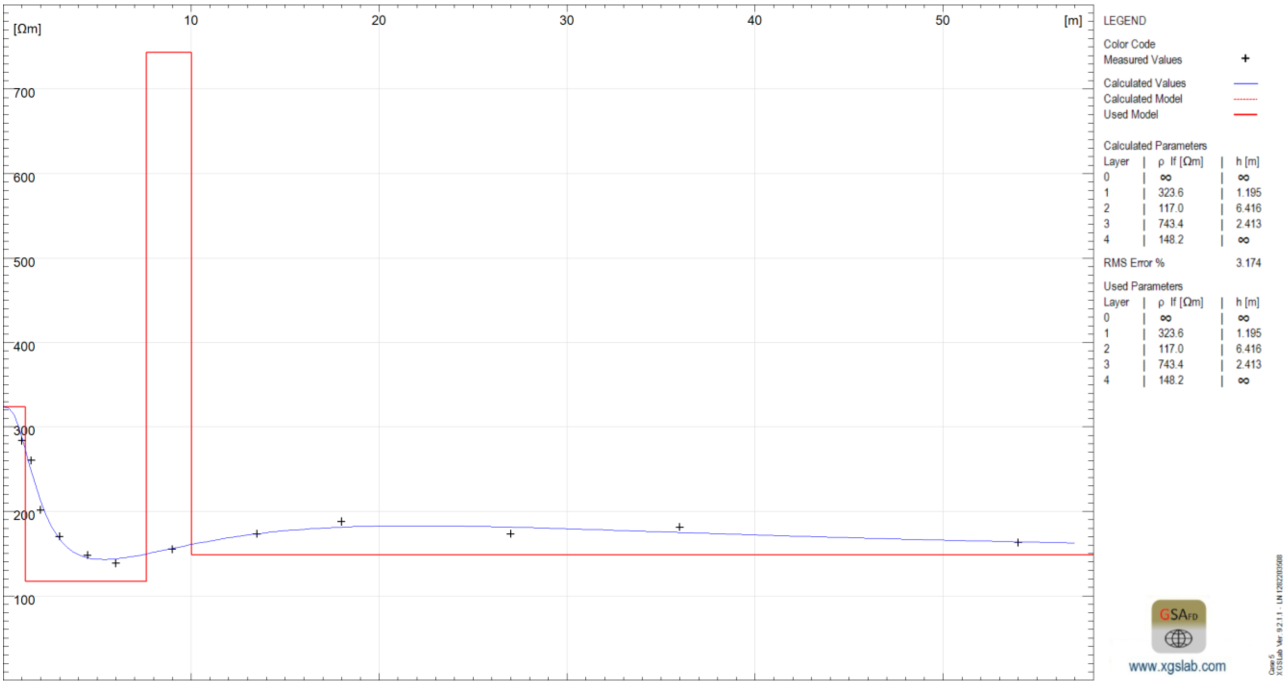

Figure 16: Soil resistivity measured values and double layers soil model

The RMS Error is quite low and is not necessary to look for a better result with more layers.

<u>Final Considerations</u>

Agreement between SRA and RESAP® is generally good but differences in results are unavoidable because with more than two layers, the search of the soil model parameters usually has not a single solution.

Moreover, solution depends also on calculation parameters as weights and constrains and sometime also on initial conditions.

The largest differences between results obtained with SRA and RESAP® are in cases when the RMS is high (more than 10%). Anyway, in these cases a higher layers number can reduce significantly the RMS Error and this opportunity should be always verified by the User.

The RMS error represents a good parameter to evaluate the solution goodness but interpreting soil resistivity requires engineering judgement and not only calculation accuracy.